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Persistence of scaling behaviour in the Game of Life

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Abstract. The persistence of scaling laws in the evolution of the automaton Game of Life from a correlated configuration is investigated. It is found that even for a highly correlated initial state, the temporal evolution is able to preserve scaling relations as observed for uncorrelated states, although with possibly discordant critical exponents. The difference can be accounted for by introducing a stochastic component in the transition rules, which at the same time establishes a connection with the possible convergence of Life to a self-organized critical state.

Early studies of the automaton Game of Life [1], or simply Life, concentrated on its ability to generate local forms of considerable diversity [2]. In fact, the particular way a local structure unfolds under Life's rules is completely unpredictable [1, 3]. Therefore, it is not possible to ascertain the offspring of even simple distributions grouped with only a few sites. More recently, the focus has shifted to a more statistical perspective where a large number of local structures are distributed on a lattice and allowed to interact for a certain period of time. In this new regime one is no longer concerned with taxonomic issues but how the large number of interacting sites evolve in time and where the evolutionary process leads the initial site distribution. From this point of view one can study the automaton either in its transient or asymptotic stage. The transient phase defines how an initial population interacts and resolves its conflicts towards some state of generally unstable equilibrium. The asymptotic phase corresponds to this quasi-equilibrium and defines how the population coexists and maintains itself. As a general rule, the transient phase is strongly dependent on the initial state, which in turn determines the nature of the asymptotic phase. Concerning the asymptotic behaviour, considerable interest has been devoted to the elusive convergence of Life to a self-organized critical (SOC) state [4]. In this case, one seeks to eliminate the role of the initial site distribution by randomizing the asymptotic state with a number of uncorrelated local perturbations. Recent experiments by Hemmingsson [5] suggest that Life does not converge to a SOC state but is in fact subcritical with a long length scale. Additional evidence has been put forward by Nordfalk and Alstrøm [6] by means of an altogether different method.

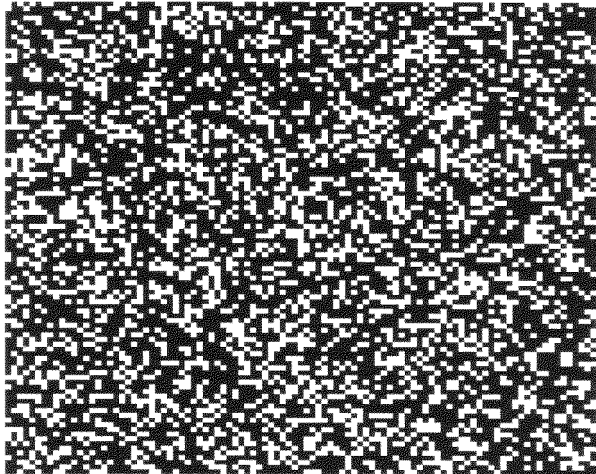
The temporal evolution of Life from some initial distribution has proven much less controversial but not the least interesting. First of all, in contrast with the study of local forms, large agglomerates exhibit novel phenomena with perhaps the most notable being the

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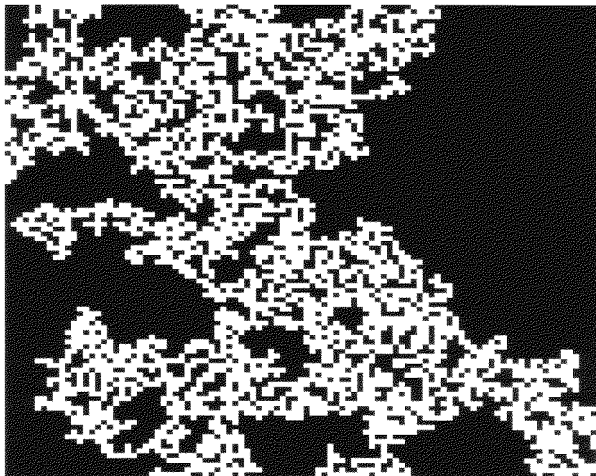
occurrence of robust temporal scaling behaviour before the system subsides into a quasi-stable configuration. This type of behaviour points to some kind of criticality although not necessarily related to SOC. Further analysis of the evolving site distribution shows [7] the presence of temporal power laws for a wide variety of geometric parameters such as the total population, diversity, mean cluster size, and so on. However, since the evolution of Life follows an intrinsically unpredictable pattern, there is some arbitrariness in the choice of an initial configuration. Because of its computational simplicity virtually all statistical studies of Life are based on uncorrelated initial states [7–10]. In analogy with the behaviour of Life from local forms, it is not clear what should happen if the initial configuration presents some degree of correlation. One might expect that, in a general circumstance, it would not be possible to predict the pattern of evolution triggered by a particular initial state. Note that we are not concerned here with the evolution of local structures but with the statistical behaviour of the aggregate and how it may depend on the initial configuration. The purpose of this paper is to show that, notwithstanding Life's unpredictability, its temporal evolution may still exhibit scaling behaviour even if the initial site configuration is nontrivial and strongly correlated. As a paradigm of a strongly correlated nontrivial site distribution we have chosen a percolation cluster (PC) [8]. In particular, we concentrate on the temporal evolution of the total population of living sites and their diversity to determine whether scaling laws are still observed and, if so, how the critical exponents compare to the well known case of random correlation.

To meaningfully compare the evolution of the correlated and the uncorrelated states, for every realization of a PC we construct another state with nearly the same mass but with the sites distributed randomly uncorrelated. This configuration is called a random-equivalent state (RES). The basic difference between the PC and its associated RES is the presence of correlation. An individual realization of these two states is shown in figure 1. The concept of a RES is useful because the general statistics of Life for random states has been extensively studied [7–10]. For a RES, the evolution of Life can be generally divided in two distinct classes of behaviour. If p denotes the probability of initial occupation, then $p > 0.15$ and $p < 0.75$ there are temporal scaling laws, otherwise no clear scaling region is observed. There are several interesting facts associated with the scaling region. After an initial phase of accommodation, temporal power-law behaviour is observed with critical exponents independent of the initial probability of occupation. Subsequently, a regime of stabilization is reached characterized by an asymptotic population of live sites also independent of p . To determine a Life configuration from a previous one the following set of rules is used: (i) a live site (state 1) with either two or three neighbours remains alive, otherwise it dies in the next generation and (ii) a vacant site (state 0) surrounded by three live neighbours becomes alive (birth). All sites are updated simultaneously with the neighbourhood of a site being defined by its eight closest sites.

We have investigated several statistical quantities including the total population or mass (number of live sites), total number of clusters, the cluster size diversity or the number of distinct cluster sizes, mean cluster size, and average site connectivity. Clusters are identified by a nearest-neighbour relation. Figure 2 shows the evolution of the mass $m(t)$ of live sites. An observed previously [9, 7], the RES presents a region of scaling with $m(t) \sim t^{-0.36 \pm 0.02}$. The PC also presents a scaling relation but with exponent -0.24 ± 0.03 . In the case of the total number of clusters the RES presents exponent -0.28 ± 0.01 while for the PC this exponent becomes -0.15 ± 0.01 . However, for all the remaining quantities the evolution of both the PC and the associated RES proceeds in a nearly identical form after the initial phase of accommodation, as illustrated in figure 3 for the cluster diversity. Consequently, the site correlation has not affected the occurrence of scaling laws, or basic functional dependence



(a)



(b)

Figure 1. Illustration of (a) a percolation cluster and (b) the associated RES, on a lattice of size 100×100 . The RES presents nearly the same mass as the percolation cluster but lacks the correlation between sites.

of any studied quantity for that matter. In addition, only for the mass and number of clusters the critical exponents have assumed different values.

This result seems to indicate that there might exist a whole class of initial states that preserves critical properties analogous to the uncorrelated case, as it happens with the percolation cluster. In the absence of external noise, this means that in an evolving environment the global dynamics would be quite insensitive to the details of spatial organization but primarily on the short-range interactions involving mutations and competition between individuals. The robustness of dynamical scaling observed in both uncorrelated and correlated configurations sheds yet another perspective on the problem of criticality in Life and allows us to establish a connection to the possibility of SOC behaviour in Life. Recent results [5] have indicated that the stabilization regime is in fact subcritical.

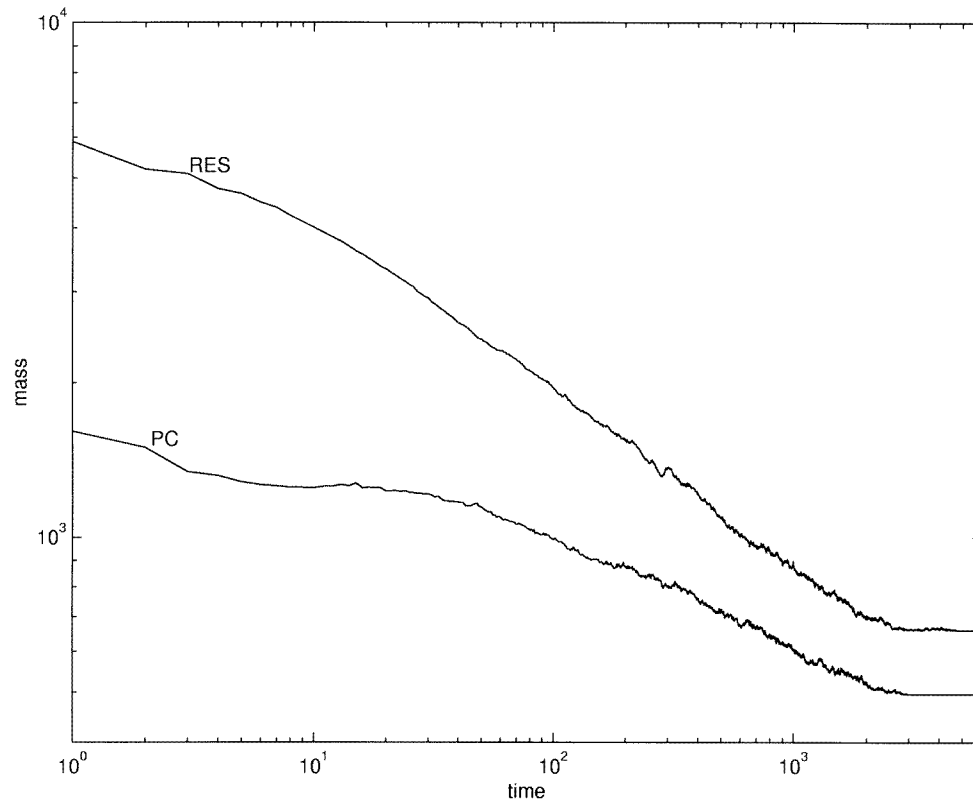


Figure 2. Temporal evolution of the total mass (sum of live sites) for a PC and the associated RES. For these plots 25 similar experiments were performed on a lattice of size $L = 150$.

However, Nordfalk and Alstrøm [6] have presented a new formulation of the problem by adding a stochastic component to the transition rules of Life. It was found that critical behaviour in the SOC sense might be observed when all the rules of Life are unchanged except that the probability of a live site surrounded by two neighbours remaining alive is no longer one but around 0.9968. The difference, although small, can have significant effects especially in the long-term limit when sparse clusters tend to be generated by the rules and individual Life structures are quite sensitive to small perturbations. This result establishes some analogies with our present results in the following sense. In order for Life to generate critical behaviour it would seem necessary to deviate from the original rules in a way that perturbs the local correlation. By perturbing the local correlation, one would restore the power-law relations typical of SOC behaviour. In our case, relating to the transient regime, local correlations are not enough to destroy critical behaviour since scaling laws are still observed. However, the critical exponents may change. Therefore, the critical exponents observed are clearly a function of the site-correlation, which seems to indicate that they do not belong to the same class of universality. Since the rules of Life do not operate at the criticality [6], we are led to question if this might be the cause for the discrepancy in the observed exponents. To test this premise we have carried experiments implementing the modified rules according to the prescription of [6]. The results are shown in figure 4. As expected, there is little effect on the first few iterations but the temporal evolution

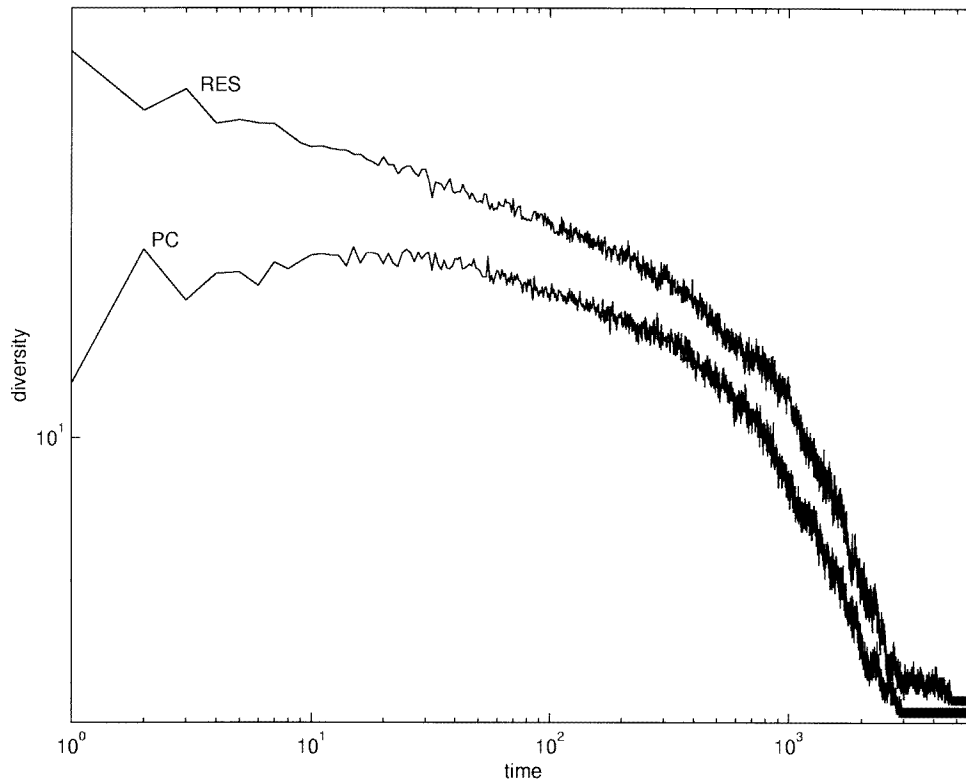


Figure 3. Evolution of the cluster diversity (number of distinct cluster sizes) for a PC and the associated RES. 25 experiments on a lattice with $L = 150$.

considerably departs from the conventional game as time tends to infinity. In particular, the scaling region is somewhat reduced but now the curves for the random case and the percolation cluster collapse after the initial accommodation and show identical exponents. In other words, when the rules of Life operate in the critical regime, the scaling properties appear to be independent of the initial correlation. In this case, the initial correlated and the random equivalent states belong both to the same class of universality. Evidently our results were limited to a single correlated state and at this point it is not possible to extrapolate our results, although they seem quite suggestive. A detailed characterization of initially correlated configurations requires further studies.

In summary, we have studied the time evolution of the automaton Game of Life from an initial correlated configuration. If Life is to be used as a model of evolutionary processes, one must consider the effect of an ecosystem where species are correlated among themselves. If the initial state presents no correlation, the presence of scaling laws and a behaviour that seems to be of relevance in the modelling of biological phenomena has been observed. It is less obvious, however, that Life could be able to retain such critical properties from an initial correlated state. We have demonstrated in the case of a percolation cluster that, even with the presence of strong site correlation, the evolution of Life still presents scaling laws. The persistence of critical behaviour extends to all statistical quantities investigated in this paper. The observed critical exponents, however, may be different from the uncorrelated case. Therefore, each set belongs to a different class of universality. This fact has led us to

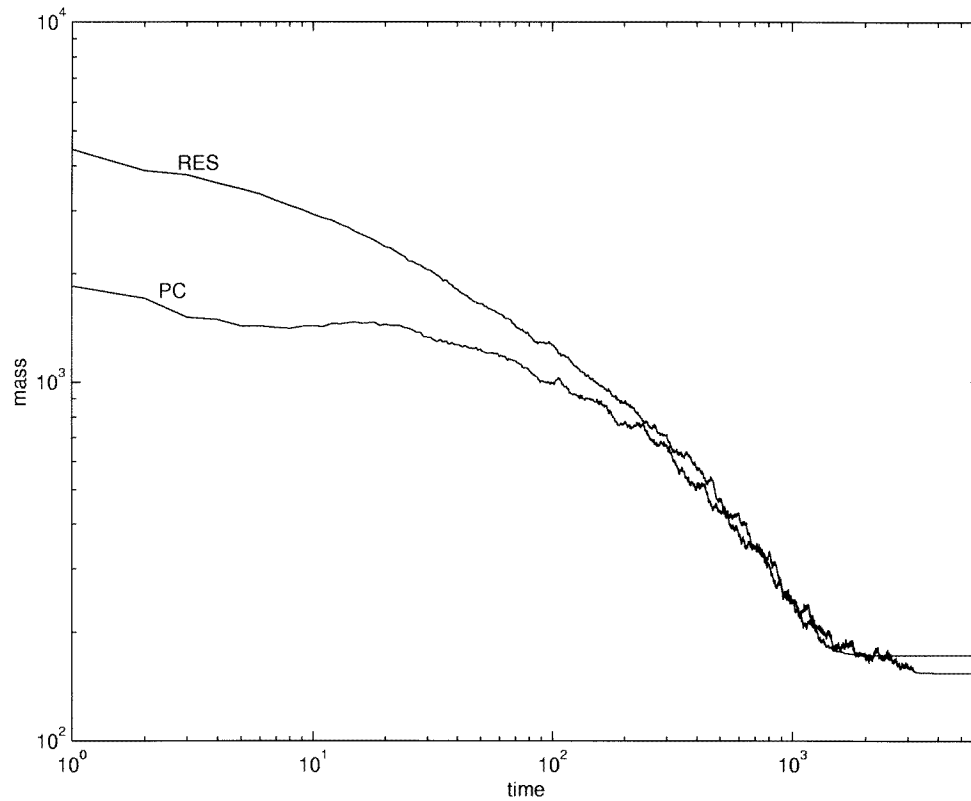


Figure 4. Temporal evolution of the mass for a PC and the RES with modified rules (see text). 25 experiments on a lattice with $L = 150$. The RES is normalized by a factor of 1.5.

conceive an unanticipated connection with the convergence of Life to a SOC state. We have shown that the modified rules that cause Life to operate in a critical condition (in the SOC sense), also cause the correlated and the uncorrelated states to belong to the same class of universality (in the transient temporal scaling sense).

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